Journal of Advanced Artificial Intelligence
Volume 1 – No.6, March 2025

Two Modulo Three Graceful Labeling of Some Special Graphs

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ABSTRACT
A function f is called a two modulo three graceful labeling of a graph G if $f: V(G) \to \{2, 5, 8, 11, 14, \ldots, 3q+8\}$ is injective and the induced function $f^*: E(G) \to \{3, 6, 9, \ldots, 3q\}$ defined as $f^*(uv) = |f(u) - f(v)|$ is bijective. A graph which admits two modulo three graceful labeling is called a two modulo three graceful graph. This paper discuss about the two modulo three graceful labeling for the graphs such as ladder graph, coconut tree, bistar graph, twig graph, regular caterpillar tree etc.

Keywords
Special Graphs

1. INTRODUCTION
Graph theory has various applications in many fields. The most important area in graph theory is graph labeling which has wide applications in computer networks, circuit design, database management and coding theory etc. A labeling of a graph $G(V, E)$ is a mapping from the set vertices or edges or both vertices and edges to the set of labels $1, 2, 3, 4, \ldots$. A function f is called a graceful labeling [8] of a graph G if $f : V(G) \to \{0, 1, 2, \ldots, q\}$ is injective and the induced function $f^* : E(G) \to \{1, 2, 3, \ldots, q\}$ defined as $f^*(uv) = |f(u) - f(v)|$ is bijective.

V. Swaminathan and C. Sekar introduced the concept of one modulo three grace- ful labeling. V. Ramachandran and C. Sekar [7] defined one modulo N graceful labeling where N is a positive integer and showed that various graphs like paths, Caterpillars, Star etc. are satisfied one modulo N graceful labeling. C. Velmurgan and V. Ramachandran [13] initiated the concept of M modulo N graceful labeling of path and star graphs [7].

C. Vimala and V. Poovila introduced the new labeling called two modulo three graceful labeling of a graph. A function f is called a **two modulo three graceful labeling** [14] of a graph G if $f: V(G) \to \{2, 5, 8, 11, 14, \ldots, 3q+8\}$ is injective and the induced function $f^* : E(G) \to \{3, 6, 9, \ldots, 3q\}$ defined as $f^*(uv) = |f(u) - f(v)|$ is bijective where q is the number of edges. A graph which admits two modulo three graceful labeling is called a two modulo three graceful graph. They proved that the comb graph, connected graph and star graph are two modulo three graceful graphs.

A. Sasikala and V. Poovila [9] discussed the two modulo three graceful labeling for bipartite graph.

2. MAIN RESULTS
Definition 2.1. A coconut tree [7] $CT(m, n)$ is the graph obtained from the path P_n by appending m new pendent edges at any one of the end vertex of P_n.

Theorem 2.2. The coconut tree $CT(m, n)$ is a two modulo three graceful graph.

Proof. Let $\{v_1, v_2, \ldots, v_{n+m}\}$ be the vertex set of $CT(m, n)$ and $\{e_1, e_2, \ldots, e_{n+m-1}\}$ be the edge set of $CT(m, n)$. We define the vertex labeling $f : V(CT(m, n)) \to \{2, 5, 8, \ldots, 3q + 8\}$ as $f(v_i) = 3i - 1, i = 1, 2, \ldots, n + m$.

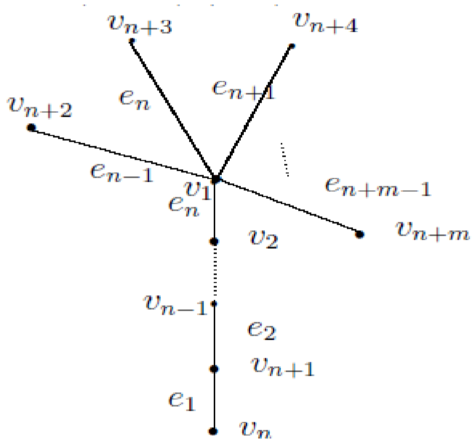

Figure 3.1

Then the induced function $f^* : E(CT(m, n)) \to \{3, 6, \ldots, 3q\}$ is defined as $f^*(v_i, v_j) = |f(v_i) - f(v_j)| = |3i - 1 - 3j + 1| = |3i - 3j| = 3|i - j|$. Hence the function $f: V(CT(m, n)) \to \{2, 5, 8, \ldots, 3q + 8\}$ is injective and the induced function $f^* : E(CT(m, n)) \to \{3, 6, \ldots, 3q\}$ defined as $f^*(uv) = |f(u) - f(v)|$ is bijective. Therefore, the graph $CT(m,n)$ admits a two modulo three graceful labeling. That is, the graph $CT(m,n)$ is a two modulo three graceful graph.

Example 2.3. The two modulo three graceful labeling of $CT(5, 4)$ is given in Figure 3.2.

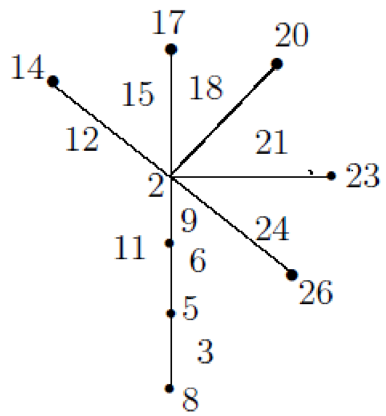

Figure 3.2

Definition 2.4. The Bistar [2] $B(m, n)$ is the graph obtained from K_2 by joining m pendant edges to one end of K_2 and

n pendant edges to the other end of K_2.

Theorem 2.5. The bistar graph $B(m,n)$ is a two modulo three graceful graph.

Proof. Let $\{v_1, v_2, \dots, v_{m+n+2}\}$ be the vertex set of $B(m,n)$ and $\{e_1, e_2, \dots, e_{m+n+1}\}$ be the edge set of $B(m,n)$. We define the vertex labeling $f : V(B(m,n)) \rightarrow \{2,5,8,\dots,3q+8\}$ as $f(v_i) = 3i-1, i = 1,2,\dots,m+n+2$.

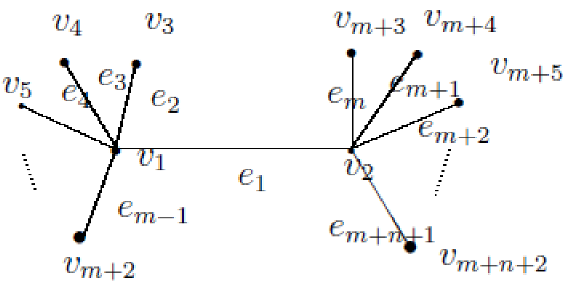

Figure 3.3

Then the induced function $f^* : E(B(m,n)) \rightarrow \{3,6,\dots,3q\}$ is defined as $f^*(v_i,v_j) = |f(v_i) - f(v_j)| = |3i-1-3j+1| = |3i-3j| = 3|i-j|$. Hence the function $f: V(B(m,n)) \rightarrow \{2,5,8,\dots,3q+8\}$ is injective and the induced function $f^* : E(B(m,n)) \rightarrow \{3,6,\dots,3q\}$ defined as $f^*(uv) = |f(u)-f(v)|$ is bijective. Therefore, the graph $B(m, n)$ admits a two modulo three graceful labeling. That is, the graph $B(m,n)$ is a two modulo three graceful graph.

Example 2.6. In Figure 3.4, we give the two modulo three graceful labeling of $B(4,4)$.

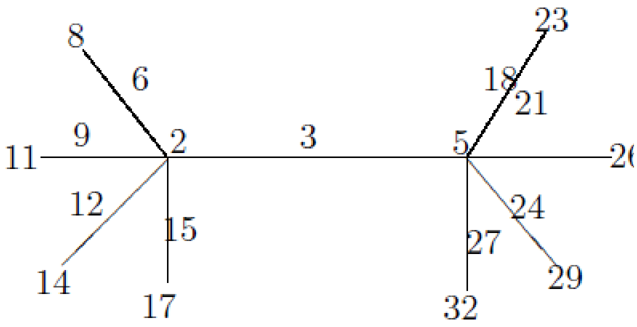

Figure 3.4

Definition 2.7. The Twig graph [6] T_m is a graph obtained from a path P_m by attaching only two pendant edges to each internal vertices of the path.

Theorem 2.8. The twig graph T_m is a two modulo three graceful graph.

Proof. Let $\{v_1, v_2, \dots, v_{2m-2}\}$ be the vertex set of T_m and $\{e_1, e_2, \dots, e_{3m-5}\}$ be the edge set of T_m. We define the vertex labeling $f : V(T_m) \rightarrow \{2,5,8,\dots,3q+8\}$ as $f(v_i) = 3i-1, i = 1,2,\dots,2m-2$.

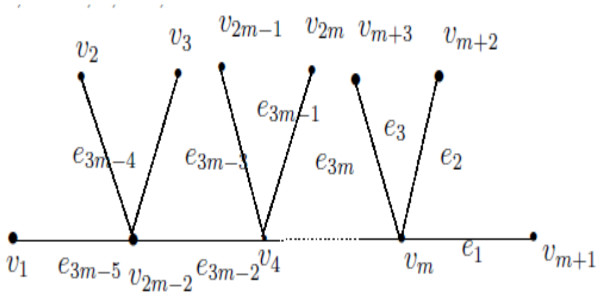

Figure 3.5

Then the induced function $f^* : E(T_m) \rightarrow \{3,6,\dots,3q\}$ is defined as $f^*(v_i,v_j) = |f(v_i) - f(v_j)| = |3i-1-3j+1| = |3i-3j| = 3|i-j|$. Hence the function $f: V(T_m) \rightarrow \{2,5,8,\dots,3q+8\}$ is injective and the induced function $f^* : E(T_m) \rightarrow \{3,6,\dots,3q\}$ defined as $f^*(uv) = |f(u)-f(v)|$ is bijective. Therefore, the graph T_m admits a two modulo three graceful labeling. That is, the graph T_m is a two modulo three graceful graph.

Example 2.9. In Figure 3.6, we give the two modulo three graceful labeling of twig graph T_4.

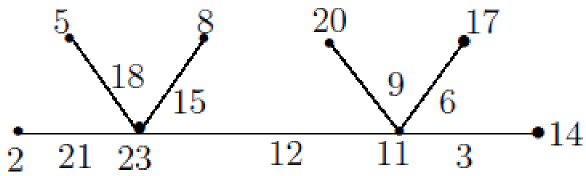

Figure 3.6

Definition 2.10. A tree is called caterpillar [3] if the removal of all its pendent vertices result in a path. If all vertices of a path have equal number of pendent vertices, then the resulting graph is called a regular caterpillar graph [6]. Regular caterpillar graph is denoted by $P_m(n)$ or $P_m \odot nK_1$.

Theorem 2.11. The regular caterpillar graph $P_m \odot 3K_1$ is a two modulo three graceful graph.

Proof. Let $\{v_1, v_2, \dots, v_{4m}\}$ be the vertex set of $P_m \odot 3K_1$ and $\{e_1, e_2, \dots, e_{4m-1}\}$ be the edge set of $P_m \odot 3K_1$. We define the vertex labeling $f : V(P_m \odot 3K_1) \rightarrow \{2,5,8,\dots,3q+8\}$ as $f(v_i) = 3i-1, i = 1,2,\dots,4m$. Then the induced function $f^* : E(P_m \odot 3K_1) \rightarrow \{3,6,\dots,3q\}$ is defined as $f^*(v_i,v_j) = |f(v_i) - f(v_j)| = |3i-1-3j+1| = |3i-3j| = 3|i-j|$.

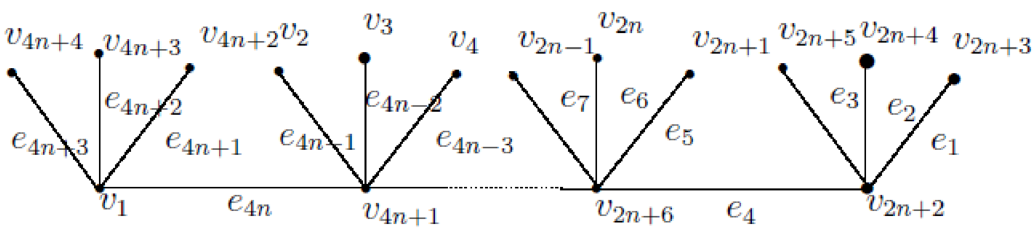

Figure 3.7

Hence the function $f : V(P_m \odot 3K_1) \rightarrow \{2, 5, 8, \ldots, 3q + 8\}$ is injective and the induced function $f^* : E(P_m \odot 3K_1) \rightarrow \{3, 6, \ldots, 3q\}$ defined as $f^*(uv) = |f(u) - f(v)|$ is bijective. Therefore, the graph $P_m \odot 3K_1$ admits a two modulo three graceful labeling. That is, the graph $P_m \odot 3K_1$ is a two modulo three graceful graph.

Example 2.12. The two modulo three graceful labeling of $P_3 \odot 3K_1$ is given in Figure 3.8.

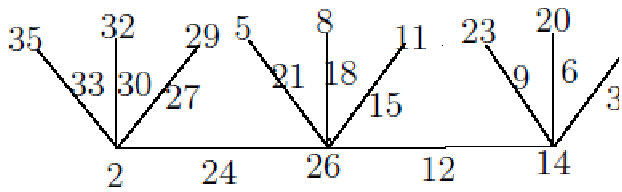

Figure 3.8

Definition 2.13. The ladder graph [5] L_n is defined by $L_n = P_n \times K_2$ where P_n is a path with n vertices, $\times$ denotes the Cartesian product and K_2 is a complete graph with two vertices.

Example 2.14. Consider the labeling of L_n is given in Figure 3.9.

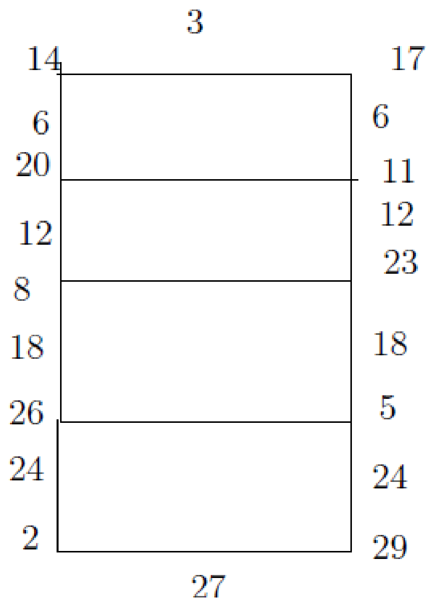

Figure 3.9

Therefore, L_n is not a two modulo three graceful graph.

Observation 2.15. The ladder graph L_n is not a two modulo three graceful graph.

Proof. If we give labels to the vertex set of L_n in a given manner, we get the edage labels which are not distinct in any one of cycles of the L_n.

3. CONCLUSION

This paper investigated the two modulo three graceful labeling of ladder graph, coconut tree, bistar graph, twig graph and regular caterpillar tree.

4. REFERENCES

[1] S.Ashok kumar and S.Maragathavalli, *Prime Labeling of Some Special Graph,* ISOR J. Math., 11 (1) (2015), 1-5.

[2] J. A. Gallian, *A dynamic Survey of graph labeling,* The Electronics J. Combi- natorics, 17 (2014), 1-384.

[3] F. Hararay and A.J.Schwenk, *The number of Caterpillars,* Discrete Math., 6(1973), 359-365.

[4] F.Haray, Graph Theory, Narosa Publishing House Reading, New Delhi, (1988).

[5] H.Hosaya and F.Harary, *On the matching properties of three fence graphs,* J. Math. Chem., 12 (1993), 211-218.

[6] N. Murugesan and R. Uma, *Super vertex gracefulness of some special graphs,* IOSR J. Math., 11(3)(2015), 7-15.

[7] V.Ramachandran and C.Sekar, *1 modulo N gracefulness of regular bamboo tree and coconut tree,* GRAPH-HOC, 6(2)(2014), 1-10.

[8] A .Rosa, *On certain valuation of the vertices of a graph,Theory of Graphs,* (International Symposium,Rome,July 1966), Gordon and Breach, N.Y. and Dunod, Paris, 1967, pp. 349-355.

[9] A.Sasikala and V.Poovila , *Two Modulo Three Graceful Labeling Of Bipartite Graph,* Adv. Appl. Math. Sci.,, 21(10)(2022), 5989 - 5996.

[10] A.Solairaju C.Vimala and A.Sasikala, *Edge-odd Graceful Labeling of the Graph* $S_2 \odot S_n$, Int. J. Comput. Appl., 9(12)(2010),

[11] P.Sumathi and A.Rathi *Quotient Labeling of Some Ladder Graph,* Amer. J. Engg. Res., (7)(12), 38-42.

[12] N.Udhayakumar and N.Ramya, *On Prime Labeling of Some Trees,* Malaya J. Matematik, (2)(2020), 4042-4045

[13] C.Velmurgan and V.Ramachandran, *M modulo N graceful labeling of path and star,* J. Inform. Comput. Sci., 9(12)(2019), 1212-1221.

[14] C.Vimala and V.Poovila, *Two Modulo Three Graceful Labeling Of Some Graphs,* Adv. Appl. Math. Sci., 21(6)(2022), 3105 - 3118.